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GRAVITY RECOVERY AND CLIMATE EXPERIMENT

UTCSR Level-2 Processing Standards Document

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(For Level-2 Product Release 0006)

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DOCUMENT CHANGE RECORD

Issue	Date	Pages	Change Description
01.0	Dec 1, 2003	All	Initial Version
01.1	July 24, 2004	6-16	Corrected TDT and GPS-time definitions in II.1.1;
			renamed sections II.1.1 and II.1.2; added
			clarifying text to II.2.1; updated and corrected
			references
02.0	Oct 5, 2005	All	Updated to RL02 background models
03.0	Feb 19, 2007	All	Updated to RL04 background models
03.1	Feb 27, 2007	8	Corrected typos in values of C30dot and C40 dot
			within the table in Section II.2.1. Added reference.
04.0	May 29, 2012	All	Updated to RL05 background models
05.0	Apr 18, 2018	All	Updated to RL06 background models

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I INTRODUCTION

I. 1 <u>Purpose of the Document</u>

This document serves as a record of the processing standards, models & parameters adopted for the creation of the Level-2 gravity field data products by the GRACE Science Data System component at The University of Texas Center for Space Research (UTCSR). This document is issued once for every release of Level-2 data products generated by UTCSR. That release number is included in the title of this document.

This document applies to the Level-2 GRACE products of the kind (see *Product Specification Document* or *Level-2 User Handbook*)

PID-2_YYYYDOY-YYYYDOY_dddd_sssss_mmmm_rrvv

Where

PID is 3-character product identification mnemonic
-2 denotes a GRACE Level-2 product
YYYYDOY-YYYYDOY specifies the date range (in year and day-of-year format) of the data used in creating this product
dddd specifies the gravity mission
sssss is an institution specific string
mmmm is a 4-character mnemonic used to identify the characteristics of the gravity solution
rrvv is a 2-digit (leading-zero-padded) release number and 2-digit (leading zero-padded) version number

This document may be used in conjunction with:

- 1. GRACE Product Specification Document (327-720)
- 2. GRACE Level-2 User Handbook (327-734)
- 3. GRACE GFZ L-2 Processing Standards Document (327-743)
- 4. GRACE JPL L-2 Processing Standards Document (327-744)
- 5. GRACE AOD1B Product Description Doc (327-750, GR-GFZ-AOD-0001)

I. 2 DOCUMENT CHANGE HISTORY

This document has been previously issued, in reverse chronological order, for the following Level-2 data product releases:

Product Release	Date Document Issued	Remarks
RL05	May 27, 2012	Doc v 4.0
RL04	February 27, 2007	Doc v 3.1

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RL02	November 4, 2005	Doc v 2.0	

RL01 July 23, 2004 Doc v 1.1

Several background gravity models have been updated since RL05. The estimated parameters have also been updated. *Please note that UTCSR Release-03 does not exist. This was done in order to synchronize release numbers with other processing centers.*

I. 3 OVERVIEW OF DATA PROCESSING

This section contains a brief overview of the data processing done to obtain the Level-2 products in this release.

The gravity field estimates were made using the conventional dynamic, linear least squares adjustment for the orbit and gravity field from an optimally weighted combination of the GPS & K-Band tracking data collected by the GRACE satellites. Some specifics follow in the next table.

Processing Institution	ocessing Institution University of Texas Center for Space Research			
	Software Used			
Orbit Software	MSODP	Version 2018.1		
Linear Solver Software	AESoP	Version 20120322_v001		
	GRACE Data Products Used	1		
Product ID & Release	Data Rate	Remarks		
ACC1B (RL=02)	1 second	Used in the numerical integrator		
SCA1B (RL=03)	1 second	For observation models & transforming body-fixed accelerations		
KBR1B (RL=03)	5-second Range Rate only			
GPS1B (RL=02)	2-minute Double Differences between one GRACE satellite, two GPS satellites & one ground station	IGS core-station network. IGb08 Combined Orbits used for GPS satellites – and held fixed during analysis		
AOD1B (RL=06)	Used as part of background gravity acceleration models			
Other Notes on Methodology				
Solution obtained as a weighted combination of GPS double differences for each satellite and inter-satellite K-Band Range-Rate – using one-day dynamic arcs over the prescribed data span. GPS data weight was limited to 2 cm for each double difference observation. K-Band range-rate was allowed optimal weighting.				
The project operational product is the outcome of the unconstrained linearized least-				

squares estimation where gravity parameters were estimated independently from the

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other orbit related parameters estimated during an earlier iteration. The regularized version of the product is experimental, and the regularization is not described in this document.

II ORBIT DYNAMICS MODELS

II. 1 EQUATIONS OF MOTION

The equations of motion for both GRACE satellites are identical in mathematical form. In the remainder of this chapter, the equations will be provided for a single Earth orbiting satellite, with the understanding that the same equations apply to both GRACE satellites. Where appropriate, the parameters or conditions unique to each satellite will be specified.

In the inertial frame

$$\overrightarrow{r} = \overrightarrow{f_g} + \overrightarrow{f_{ng}} + \overrightarrow{f_{emp}}$$

where the subscript "g" denotes gravitational accelerations; "ng" denotes the acceleration due to the non-gravitational or skin forces; and "emp" denotes certain empirically modeled forces designed to overcome deficiencies remaining in the force models.

II.1.1 Time Systems

The independent variable in the equations of motion is the time system TDT (Terrestrial Dynamical Time). The relationship of this abstract, uniform time scale to other time systems is well defined. The table below shows the relationship between various time systems and the contexts in which they are used.

System	Relations	Notes	Standards
TAI	Fundamental time	International Atomic Time	n/a
	system		
UTC	UTC = TAI - n1	n1 are the Leap Seconds	Tables from USNO
	(Time-tag for saving		
	intermediate products)		
TDT	TDT = TAI + 32.184s	This is the independent	IAU 1976
		variable for numerical	Recommendation
		integration.	(equivalent to using
			TT in <i>IERS-2010</i>)
GPS	GPS = TAI - 19s	Relationship between GPS	Time-tags in sec
	(time-tag of GRACE	& TDT is fixed at 19	since 1200 Jan 01,
	observations)	seconds	2000 GPS Time.

II.1.2 Reference Frames

The fundamental reference frame for the mathematical model is the non-rotating, freefalling (inertial) reference frame, with the origin defined as the center of mass of the Earth system. The Inertial and Earth-fixed reference frames, and their relative orientations and associated standards are further described in the chapter on Earth Kinematics.

II. 2 GRAVITATIONAL ACCELERATIONS

The gravitational accelerations are the sum of direct planetary perturbations and the geopotential perturbations. The vector of direct planetary perturbations is evaluated using the planetary ephemerides. All geopotential accelerations are represented using a spherical harmonic expansion with time-variable coefficients, to a specified maximum degree and order. The accelerations are computed by evaluating the Earth-fixed gradient of the geopotential, which are then rotated (after summation with the non-gravitational accelerations) to inertial frame for the integration of equations of motion. In general,

$$\vec{f}_{g} = _{3x3} M_{ef}^{in}(P, N, R) \vec{f}_{g}^{ef}$$

The 3x3 rotation matrix M, which depends on Earth orientation is described in the chapter on Earth Kinematics.

Contributions to the spherical harmonic coefficients of the geopotential, and the associated implementation & standards are now compiled. The geopotential at an exterior field point, at time t, is expressed as

$$U_{s}(r,\varphi,\lambda;t) = \frac{GM_{e}}{r} + \frac{GM_{e}}{r} \sum_{l=2}^{N_{max}} \left(\frac{a_{e}}{r}\right)^{l} \sum_{m=0}^{l} \overline{P}_{lm}(\sin\varphi) \left[\overline{C}_{lm}(t)\cos m\lambda + \overline{S}_{lm}(t)\sin m\lambda\right]$$

where r is the geocentric radius, and (φ, λ) are geographic latitude and longitude, respectively, of the field point.

The suite gravitational models used for propagation of the equations of motion of the satellites are called the Background Gravity Models. This concept, and its relation to GRACE estimates, is described further in the *Level-2 User Handbook*. The details of the background gravity model are provided here.

Hereafter, the document *IERS Conventions (2003)* is abbreviated as *IERS-2003*, and the *IERS Conventions (2010)* as *IERS-2010*.

Parameter	Value	Remarks		
GM _e	3.986004415E+14	<i>IERS-2010 Standards (value is consistent with using TDT or TT as the time argument)</i>		
a _e	6378136.3 m			
N _{max}	Complete to degree and order 360	<i>GGM05C</i> is background static model.		
Secular Change	N/A	Not modeled in background.		
Note 1: The normalization conventions are as defined in <i>IERS-2010</i> , Eqs 6.2-6.3.				
Note 2: The implementation of computation of spherical harmonics and its derivatives is				

II.2.1 Mean Geopotential & Secular Changes

as described in (Lundberg and Schutz, 1988).

Note 3: The degree-1 terms are exactly zero in the geopotential model.

<u>Note 4:</u> The mean field *GGM05C* is a mean gravity field model created from a combination of approximately 10 years of RL05 products spanning from March 2003 through May 2013. The GRACE data were combined with GOCE gradiometer data from November 2009 to October 2013. In addition, harmonic coefficients extracted from the DTU13 gravity anomaly dataset extended GGM05C to degree and order 360, as described in *Ries et al. (2016)*. The model coefficients are available from GRACE data archives, ICGEM, and from ftp://ftp.csr.utexas.edu/pub/grace/GGM05

II.2.2 Solid Earth Tides

Solid Earth tidal contributions to the geopotential are computed for the an-elastic Earth model, as specified in Section 6.2, *IERS Conventions (2010)*. Corrections to specific spherical harmonic coefficients are computed and added to the mean field coefficients.

Model	Notes	
Planetary Ephemerides	DE-430	
	Degree 2 & 3 – expression in	Parameter values from Table
Frequency Independent	Eq 6.6, <i>IERS-2010</i> .	6.3
Terms (an-elastic	Ellipticity contributions from	As per Eq. 6.7, <i>IERS-2010</i>
Earth)	Degree 2 tides to Degree 4	
	terms	
Frequency Dependent	Corrections to all degree-2	As per Tables 6.5, <i>IERS-2010</i>
Terms	terms	
		Subtracted from total
Permanent Tide in \overline{C}_{20}	4.201E-9	contributions as calculated
(zero-tide system)	T.201L-7	above (implicitly included in
		value of the mean C20)

II.2.3 Ocean Tides

The ocean tidal contributions to the geopotential are computed as specified in the prefatory material for Section 6.3, *IERS-2010*. Corrections to specific spherical harmonic coefficients of arbitrary (selectable) degree and order are computed and added to the mean field coefficients. The background ocean tide models are detailed in the table below, and the models and the methods of interpolation to minor tidal constituents are different from the model and methods specified in *IERS-2010*.

Model	Description	Notes
Tidal Arguments &	Doodson (1921)	
Amplitudes/Phases	Cartwright & Tayler (1971)	
	Harmonics of model GOT4.8	
Diurnal/Semi-Diurnal	(<i>Ray 2012, pers. comm.</i>) to	
Bands	degree 180 (See Note-1)	
	Periods > Monthly: Self-	Extended to all minor
	consistent equilibrium model	constituents by fitting

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Long-Period Band	(Ray 2005 pers. comm. based on Ray & Cartwright 1994)Mm and Mf: Egbert & Ray 2003Mtm and Msm: FES2004 (Lefevre 2005)	admittances to the provided estimates for the major tides.	
Note-1: GOT4.8 differs from GOT4.7 in only the harmonics of the S2 tide.			

The implementation is as follows. The contributions to C_{lm} and S_{lm} values from all the lines in the Cartwright & Tayler expansion are pre-computed and saved in data files for each calendar day at 10-minute intervals. During orbit processing, the software reads these data files, and interpolates the contributions to the acceleration evaluation epoch. Further details are available in *Bonin 2005*. The ocean tide model used in RL06 is the same as the one used in RL05.

II.2.4 Atmosphere & Oceanic Variability

The non-tidal variability in the atmosphere and oceans is removed through using the AOD1B Release-06 product. It is based on analysis and forecast data of the operational high-resolution global numerical weather prediction (NWP) model from the European Centre for Medium-Range Weather Forecasts (ECMWF) and ocean bottom pressure from an unconstrained simulation with a global ocean general circulation model that is consistently forced with ECMWF atmospheric data. The details of this product and its generation are given in the *AOD1B Description Document (GRACE 327-750)*. ftp://podaac-ftp.jpl.nasa.gov/allData/grace/docs/AOD1B_PDD_RL06_v6.0.pdf

This model of the geopotential is available as 3 hourly time series to degree and order 180. The value of the harmonics at intermediate epochs is obtained by interpolation between the bracketing data points.

In order to improve the accuracy of interpolation, the following procedure is adopted. First, the 3-hr epoch values from the AOD1B product are interpolated to 10-minute intervals called the "AOT" product. The **Ray/Ponte** S_2 air tide model is also evaluated at 10-minute intervals and added back to the "AOT" product because the AOD1B RL06 does not include the S2 tide.

Just as with the ocean tides, the software reads these data files during orbit integration and interpolates the contributions to the acceleration-evaluation epoch.

II.2.5 Solid Earth Pole Tide (Rotational Deformation)

The rotational deformation forces are computed as additions to spherical harmonic coefficients \overline{C}_{21} and \overline{S}_{21} , from an elastic Earth model, as specified in Section 6.4, *IERS*-2010.

Model	Description	Notes
An-Elastic Earth Model	Scaled difference between epoch pole position and mean	
Contribution to C21 & S21	pole. See Chapter III (Earth Kinematics) for the linear	

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	variation model for the mean pole.	
Polar Motion	Tabular input	IERS C04
Mean Polar Motion	Linear model	IERS-2010 ⁻¹

II.2.6 Ocean Pole Tide

The self-consistent equilibrium model of **Desai** is used (*Desai 2002*). A spherical harmonic expansion to degree 180 is used, with the same polar motion time series as for Earth Kinematics or the Solid Earth Pole Tide (See Section II.2.5).

The contributions to the spherical harmonic coefficients are pre-computed using software provided (*http://tai.bipm.org/iers/convupdt/convupdt_c6.html*), and stored at 10-minute intervals for each calendar day. The orbit processing software reads these data files and interpolates the contributions to the integration or the acceleration evaluation epoch.

II.2.7 N-Body Perturbations

Unlike the geopotential accelerations, the perturbations due to the Sun, Moon and all the planets are directly computed as accelerations acting on the spacecraft. The direct effects of the objects on the satellite are evaluated using point-mass attraction formulas. The indirect effects due to the acceleration of the Earth by the planets are also modeled as point-mass interactions. However, for the Sun and Moon, the indirect effects include, in addition, the interaction between a point-mass perturbing object and an oblate Earth – the so-called Indirect J2 effect.

Model	Description	Notes
Third-Body Perturbation	Direct & Indirect terms of point-mass 3 rd body	
	perturbations	
Indirect J2 Effect	Sun & Moon only	
Planetary Ephemerides	DE-430	

II.2.8 General Relativistic Perturbations

The general relativistic contributions to the accelerations and light propagation are computed as specified in Chapters 10 and 11 of the *IERS-2010 Conventions*.

II. 3 NON-GRAVITATIONAL ACCELERATIONS

The nominal approach is to use the GRACE accelerometer data to model the nongravitational accelerations acting on the satellite. The model used is:

$$\vec{f}_{ng} = q \otimes \left[\vec{b} +_{3x3} \mathbf{E} \; \vec{f}_{acc} \right]$$

where the q/operator represents rotations to inertial frame using the GRACE Attitude Quaternion SCA1B product; b represents an empirical bias vector; and the 3x3 diagonal

¹ See update at http://maia.usno.navy.mil/conventions/2010/2010_update/chapter7/

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matrix E contains the scale factors. The bias vector and scale matrix operate on the GRACE Accelerometer observation ACC1B product, and are estimable parameters.

II. 4 EMPIRICAL ACCELERATIONS

No empirical accelerations, either mean or once-per-revolution, were estimated.

II. 5 NUMERICAL INTEGRATION

The Predictor-Corrector formulation of the Krogh-Shampine-Gordon, second order, fixed-step, fixed-mesh/order integrator is implemented.

Model	Description	Notes
Dependent Variables		
1. Equations of motion (position/velocity for each satellite)		
2. State Transition Matrix (position/velocity mapping terms only)		
Formulation	Cowell Formulation	
Step-size and Order	5 seconds and 7 th order	

III EARTH & SATELLITE KINEMATICS

III. 1 EARTH ORIENTATION

Earth Orientation here refers to the model for the orientation of the Earth-fixed reference relative to the Inertial reference. The former are necessary for associating observations, models and observatories to the geographic locations; and the latter for dynamics, integration & ephemerides.

Frame	System	Realization
Inertial	ICRS	J2000.0 (IERS-2010)
Earth-fixed	CTRS	IGS2008

The rotation between the Inertial and Earth-fixed frames is implemented as:

$_{3x3}M_{trs}^{crs} = Q(t)R(t)W(t)$

which converts the column array of components of a vector in the terrestrial frame to a column array of its components in the inertial frame. Each component matrix is itself a 3x3 matrix, and is now individually described.

III.1.1 Precession & Nutation

Precession and Nutation are modeled using IAU2000A model (*Capitaine et al. 2002, Mathews et al. 2002*). Reference epoch 2000.0 is used. The independent variable is TT since epoch J2000.0 (noon, 01-Jan-2000).

III.1.2 Sidereal Rotation

This rotation is implemented as

$$R = R_3(-GST)$$

where the Greenwich Apparent Sidereal Time (GST) is calculated as the sum of Greenwich Mean Sidereal Time (GMST) and equatorial components of precession and nutation, consistent with the IAU 2000 resolutions. The GMST calculation uses UT1 as its independent argument, whose evaluation is also summarized in this table.

Quantity	Model	Notes
GMST	Polynomial with UT1 as	IERS-2003 Standards
	independent variable.	
Equatorial components of	(Aoki & Kinoshita 1983)	IERS-2003 Standards
precession & nutation		
	cubic interpolation of	IERS C04
	tabular UT1 corrections	
UT1	Diurnal tidal variations	IERS-2010 Standards
	from Ray et al., 1994 eight	
	constituent model.	

III.1.3 Polar Motion

The Polar Motion component of rotation is implemented as

$$W = R_1(y_p)R_2(x_p)$$

Quantity	Model	Notes
Tabular variations	Cubic interpolation	IERS C04
Ocean Tidal Variations	Orthoweights Formulation	IERS-2010 Standards
(Diurnal/Semi-Diurnal)		
Note 1: The rotation matrices are implemented in the small angle, skew-symmetric		

matrix formulation.

Note 2: Rotational deformation accelerations & kinematic station displacements are proportional to the difference between this time-series and a cubit mena-pole model.

III. 2 STATION COORDINATES

This section summarizes the models for the mean and time-variable parts of the station coordinates adopted for data processing.

Quantity	Model	Notes
Mean Station Positions and Velocities	IGb08 (epoch-specific values are used)	Refers to the position of a geodetic marker or instrument reference point at each site.
Ocean Tidal Loading (Diurnal/Semi-Diurnal band)	Tidal orthoweights adjusted to site dependent displacement coefficients from Scherneck's loading service site.	<i>IERS-2010</i> Table 7.4, using GOT4.7 ocean tide model
Station Eccentricities	Individual observation model	S
Luni-Solar Solid Earth Tidal displacement	Chapter 7, <i>IERS-2010</i> (Luni-Solar ephemerides from DE-430)	
Rotational Deformation	Scaling of difference of polar motion values from a cubic trend model.	Will switch to linear mean pole when a new IGS14 realization is produced consistent with that model.
Tidal Geocenter (Diurnal/Semi-Diurnal)	Included within Ocean Tidal Loading model	
Atmospheric Loading	Not modeled	No conventional model
Post-glacial Rebound	Not modeled	No conventional model
Seasonal Geocenter Variations	Not modeled	No conventional model

III. 3 SATELLITE KINEMATICS

The Science Reference Frame (SRF - see *Product Specification Document*) is used in all instances where a satellite-fixed reference frame is needed. The inertial orientation of the spacecraft (i.e. the SRF) is modeled using tabular input quaternion from product SCA1B (*ibid.*). The same product is also used for rotating the accelerometer data to inertial frame prior to numerical integration; for making corrections to the ranging observations due to offset between the satellite center of mass & the antenna location; as well as for computing the non-gravitational forces (if necessary).

At epochs where the GRACE quaternion product is not available, linear interpolation between adjacent values is used.

III.3.1 Rotation of Velocity Components

The position rotations are specified in Section III. 1. The components of the satellite velocity vector are rotated using the matrix approximation

$$\overset{\sqcup}{v_{crs}} = M_{crs}^{trs} \overset{\sqcup}{v_{trs}} + (PN\dot{R}S) \overset{\sqcup}{r_{trs}}$$

III.3.2 GRACE GPS Antenna Offset Model

The GRACE GPS navigation receiver is placed on the top surface (see *Product Specification Document*). For the purposes of orbit and gravity field determination, the antenna phase center location vector for the L3 (LC) ionosphere-free double difference is

(0.0, 0.0, -0.419) meters (TBD)

in the Science Reference Frame. This is consistent with the value provided in the VGN1B product (RL02). The IGb08 antenna phase-center variations (PCV) maps for the GPS transmitter satellites and for the ground-stations were modeled, as were the GRACE receiver antenna PCV maps.

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